Combustion of sprays in an oxidizing atmosphere

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SUMMARY

1. - The Mathematic representation of the statistical size distribution of droplets injected into the combustion chamber of a Diesel engine and a gas turbine, is discussed.

After consideration is given to some of the distributions proposed for other Technical Sciences (Meteorology - Gravimetrology, etc.), the Mugele Evans distribution is selected as the most adequate for the problem.

*2. - The combustion of a spray under ideal conditions is analyzed. It is assumed for it that each droplet burns in accordance with a theoretical law whose validity has been experimentally verified for a few simple cases - isolated droplets at rest or in motion, set of droplets in motion whose relative positions remain invariable, etc.**

The particular cases considered are the following: Combustion at the beginning and end of the injection. Combustion at an instant sufficiently far in time from both the above cases. Combustion of a volume injected instantaneously as approximately occurs in a Diesel engine and, finally, super-position of isolated successive injections.

3. - The scaling of atomizers and combustion chambers is analyzed.

Combustion de jets pulvérisés dans une atmosphère oxydante

RÉSUMÉ

1. - On discute ici la représentation mathématique de la distribution statistique des grandeurs de gouttelettes injectées dans la chambre de combustion d'un moteur Diesel et d'une turbine à gaz.

Après avoir considéré quelques distributions proposées pour d'autres Sciences Techniques (Météoro-logie, Gravimétrologie, etc.), on a choisi la distribution Mugele Evans comme la plus appropriée pour le problème en question.

*2. - On analyse la combustion d'un jet en conditions idéales. On envisage pour cela que chaque gouttelette brûle selon une loi théorique dont la validité a été vérifiée expérimentalement pour quelques cas assez simples, gouttelettes isolées en mouvement ou non, ensemble de gouttelettes en mouvement dont les positions relatives restent invariables, etc.**

Les cas particuliers considérés sont les suivants: Combustion au début et à la fin de l'injection. Combustion dans un instant assez éloigné en temps des deux cas ci-dessus. Combustion d'un volume injecté instantanément ainsi qu'il se produit à peu près dans un moteur Diesel et, enfin, super-position d'injections isolées successives.

3. - On analyse également la similitude dynamique des injecteurs et des chambres de combustion.

Über die Verbrennung von Kraftstoffstrahlen in einer oxydierenden Atmosphäre

ZUSAMMENFASSUNG

1. Es wird hier die mathematische Darstellung der zahlenmäßigen Verteilung von Tröpfchengrößen diskutiert, welche in die Verbrennungsräume eines Dieselmotors und einer Gasturbine eingespritzt werden.

Nach Behandlung von Verteilungen, welche für andere Zweige der Wissenschaften (Meteorologie, Gravimetrologie usw.) in Betracht kommen, wird die Zerstäubung „Mugele Evans-Verteilung" als die am besten geeignete für das zur Diskussion stehende Problem vorgeschlagen.

*2. Die Verbrennung eines Einspritzstrahles unter idealen Bedingungen wird analysiert. Hierfür unterstellt man, daß jedes Tröpfchen nach einem theoretischen Gesetz verbrennt. Die Gültigkeit dieses Gesetzes ist für einige einfache Fälle experimentell erwiesen: Einzelne Tröpfchen in Be-wegung oder in Ruhe; eine Tropfenschar in Bewegung, wobei die Lage der Tröpfchen zueinander unverändert bleibt, usw.**

Die in Betracht gezogenen besonderen Fälle sind: Verbrennung zu Beginn und am Ende der Ein-spritzung: Verbrennung in einem Augenblick, der von beiden obigen Fällen zeitlich ausreichend getrennt ist: Verbrennung eines plötzlich eingespritzten Volumens, wie es sich ungefähr beim Diesel-motor abspielt, und schließlich die Überlagerung einzelner, aufeinander folgender Einspritzungen.

3. Die Ähnlichkeitsverhältnisse von Einspritzdüsen und Verbrennungsräumen wird ebenfalls analysiert.

CONTENTS

1. – Drop Size Distribution in Sprays

1.1. – In the majority of the processes of technical interest substances are used which, being liquid in normal conditions, burn in gaseous phase. For this it will be necessary for the liquid to evaporate before beginning the combustion. As the mass transfer from the liquid phase to the gaseous phase is a function of free surface, it will be desirable to increase the said surface as far as possible; this can be done by atomizing the liquid introduced into the combustion chamber.

The first theoretical problem which presents itself is that of defining the spray as a cluster of droplets for which one tries to classify these by their size, using for this purpose a distribution function F that represents, *for each size*, the relation between the volume corresponding to all the droplets of inferior size to that under consideration, and the total volume.

1.2. – Different attempts have been made, amongst which we will mention those corresponding to references (1) and (2), tending to approach the size distribution of th: particles of a spray, by means of formulas obtained from experiments.

The Rosin Rammler distribution was proposed, in principle, to be applied to the study of powdered materials but it has been used extensively in processes of atomization because it is very simple.

Normally, it is found in the form:

$$F(x) = 1 - e^{-\left(\frac{x}{\bar{x}}\right)^{\delta}}$$

in which x is a parameter of size and δ is a parameter of distribution. The greater is $\bar{x}$ the greater is the mean diameter of the droplets. The greater is δ, the nearer are the sizes of the particles to the mean diameter.

The Nukiyama Tanasawa distribution, obtained from experiments, is given in the formula:

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$$F(x) = \frac{\Gamma_{bx\delta}\left(\dfrac{6}{\delta}\right)}{\Gamma\left(\dfrac{6}{\delta}\right)}$$

being $\Gamma_{bx\delta}\left(\dfrac{6}{\delta}\right)$ the incomplete factorial function defined by means of the expression:

$$\Gamma_y(x) = \int_0^y t^{x-1} e^{-t}\, dt$$

$b^{-\frac{1}{\delta}}$ and δ are the parameters equivalent to $\bar{x}$ and δ in the Rosin Rammler distribution.

1.3. - A completely different way of finding the distribution function sought, is that taken by Mugele and Evans (3). Their point of view, which is more mathematical than experimental, may be summarised in the following manner: Taking into account that the process of atomization is a random one, it is logical to think that the appropriate election of an independent variable may convert the distribution of volumes into a gaussian distribution. It has been proved by experiment that this is nearly exact if one takes

$$y = \ln a\, \frac{x - x_0}{x_{max} - x}$$

as an independent variable.

If one considers a gaussian distribution on y, a distribution function on x will result in the following form·

$$F(x) = {}^1\!/_2 \left[1 + \varPhi \left(\delta \ln a\, \frac{x - x_0}{x_{max} - x} \right) \right]$$

which is the formula given by Mugele and Evans. In it δ is the parameter of distribution, x_{max} and x_0 are the maximum and minimum diameters respectively and $1/1 + a$ defines the diameter of the droplets for which $F(x) = {}^1\!/_2$.

$\varPhi(x)$ is the error function.

As follows we shall use the Mugele and Evans function in which we suppose that $x_0 = 0$.

The advantages of this distribution function and the remarkable results obtained (3) in the statistical representation of the sprays is due to the fact that, by depending the function on three parameters δ, a and x_{max}, it is possible to adjust it more exactly than in any other known distribution to the experimental data. On the other hand, the Mugele Evans formula makes it possible to take into account the fact that the maximum size of the droplets obtained in the spray is finite.

1.4. - In Table I the three distributions referred to above are compared. In it values of the characteristic parameters of the three formulas (taking a = 1 on that of Mugele-Evans) are given for distributions in sprays of liquid with surface tensions of the order of 300 dynes/cm, absolute viscosities comprised between 0,020 and 0,0040 poises and pressures of injection of about 100 kg/cm². The distributions have been compared taking equal values of the Sauter diameter $\bar{x}_{32}$ and the diameter corresponding to 50% of the volume of the spray, x_{50}.

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TABLE I

Mugele Evans		Nukiyama Tanasawa		Rosin Rammler			
a	δ	b	δ	$\bar{x}$	δ	$\bar{x}_{32}$	x_{50}
0.50	0.50	29.52	0.22	.813	1.85	.424	.667
0.50	1.	6.82	1.50	.721	4.70	.609	.667
1	0.50	44.08	0.15	.628	1.61	.269	.5
1	1.	11.31	1.04	.561	3.25	.438	.5
1.50	1.	14.79	0.85	.448	3.25	.342	.4

The figures 1 to 5, represent the values of $F(x)$ versus $\xi = \dfrac{x}{x_{max}} \leq 1$ in the cases corresponding to the distributions of the foregoing table.

The great influence of the droplets of large diameter on the value of $\bar{x}_{32}$, will be seen, which explains that for ξ near to one, the Mugele-Evans distribution gives greater values of F than the other two.

2. – Combustion of Sprays

2.1. – The combustion of sprays is a very complicated phenomenon, in which the processes of atomization and evaporation, mixing and chemical kinetics are co-existent. A sufficiently complete knowledge of the phenomenon can be obtained by trying to study separately each one of the partial processes.

In this paragraph we shall deal exclusively with the two last processes, mixing and chemical kinetics.

2.2. – Experiments made on the combustion of isolated droplets of different fuels have shown the existence of a linear law of variation of the surface of the droplet with the time. More extensive theoretical studies, which are, at present, being made at I.N.T.A., seem to show that the said law is valid only if the activation energy of the chemical reaction is very great, as in the case of the usual hydrocarbons and fuels.

The supposition that the activation energy is great, or, which amounts to the same thing, that the time necessary to complete the combustion is very short compared to the characteristic time of the global process, allows one to suppose that the chemical kinetics has little influence and that the problem is reduced to one of diffusion alone. Therefore, the use of the words evaporation and combustion from now on is justified.

2.3. – An attempt has been made to study the combustion of sprays in the following form: It is supposed that the diameter of each droplet varies according to the law

$$X^2 = X_o^2 - \lambda t$$

in which the constant of evaporation λ is a function of the velocity of the droplet, of the temperature and, therefore, of the proximity to the flame fronts of the surrounding droplets and, finally, of the initial size of the droplets. Starting from a droplet size distribution of the

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injected droplets, the evolution of the cluster can be studied mathematically, taking a mean value of the evaporation constant.

The first attempt to study the combustion of a spray in such form is due to Probert (4), who used the statistical formula of Rosin Rammler and studied the distribution of the sizes of the existing droplets in the chamber, on the supposition that the process of injection is instantaneous.

2.4. – Unfortunately, it has not been possible so far to make an experiment that will throw sufficient light on the validity or non-validity of such theoretical procedure in the study of the process of combustion of a spray.

Graves and Gerstein (5) compare the efficiency obtained in a certain type of burner with the efficiency calculated in accordance with Probert's method, using a mean evaporation constant on an isolated droplet. The results seem to be descouraging enough. However, it is clear that a motionless droplet, isolated and situated in an atmosphere at rest, cannot evaporate in accordance with the same law as a droplet in movement, in which the flame front approaches the droplet due to the influence of the convection, that is in the proximity of other droplets the flame fronts of which activate the combustion of the droplet under consideration.

In order to study the influence of these factors, experiments have been made with nearer droplets suspended from quartz fibres and situated in an atmosphere at rest, following, photographically, the life of a droplet enclosed in an array of others that are burning (6), (7), (8).

In the same way drops have been suspended in an airflow in order to measure the effect of forced convection (9).

Ingebo (10) made several successful experiments recording photographically, from different fixed stations, a spray in evaporation and using, between each two stations, evaporation constants based on the ones corresponding to the droplets of mean diameter of each station and taking into account the influence of forced convection by means of a law of the form proposed by Frössling.

2.5. – However, the definite experiment of measuring droplet size distribution during the process of evaporation of a spray the initial statistical distribution of which is known, has not so far been made.

In this work an experimental procedure to check the validity of the Probert method is proposed, consisting in eliminating the additional difficulty which introduces the initial distribution of the particles in sizes, using a method of atomization by means of a rotating cup which under certain conditions (11) produces sprays initially uniform and obtaining the distribution of the existing particles in every instant by photographing the zone of the spray.

The fundamental advantage of this method is that the droplets being initially equal, their especial distribution is independent of the time and the distributing effect due to the influence of the aerodynamic forces on them.

The reader who desires to consider other models of combustion of droplets different from the "static" one under consideration here, may consult references (12) and (13).

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2.6. – In the paper presented to the *International Congress of Combustion Engines* (14) celebrated at Zurich in 1957, we considered the combustion of a spray the injection of which is steady (15).

This study idealized the existing conditions in the interior of the combustion chamber of a gas turbine at instants very far from the beginning of the injection as well as from the end.

In the present chapter the previous results are generalized taking into consideration the combustion at instants near the beginning or the end of the injection and studying the combustion of sprays injected instantaneously. The first two cases correspond to the starting and stopping of a gas turbine and the last to the functioning of a Diesel engine.

2.6.1. – *Volume which has been burned in every instant in the period of initiation of the combustion.*

If G is the volume injected in the unity of time, the total value burned t seconds after the beginning of the injection is given by the following expression (15).

$$g_i = \left[\tau - \frac{\delta}{\sqrt{\pi}} (I - I_1) \right] G t_v$$

in which: t_v is the lifetime of the largest droplets of spray.

$$\tau = \frac{t}{t_v}$$

$$I = \int_0^1 x^4 \, dx \int_0^{1-x^2} \frac{\exp\left[-\left(\delta \ln \frac{a \, (x^2 + \tau)^{1/s}}{1 - (x^2 + \tau)^{1/s}} \right)^2 \right]}{(x^2 + \tau)^{5/s} \, (1 - (x^2 + \tau)^{1/s})} \, d\tau$$

$$I_1 = \int_0^1 u \, (1 - x^2 - \tau) \, x^4 \, dx \int_0^{1-x^2} \frac{\exp\left[-\left(\delta \ln \frac{a \, (x^2 + \tau)^{1/s}}{1 \, x^2 + \tau)^{1/s}} \right)^2 \right]}{(x^2 + \tau)^{5/s} \, (1 - (x^2 + \tau)^{1/s})} \, d\tau$$

u is the unit-step function

$$u \, (y) = 1 \qquad y > 0$$
$$u \, (y) = 0 \qquad y < 0$$

From the foregoing expressions it is deduced that if $\tau \geq 1$, the volume burned in each instant is independent of the time and its value is precisely that calculated in the case of a steady injection (14). Therefore, the result is that, if we suppose the conditions which influence the injection of the fuel to be invariable, the duration of the transitory period of initation of the combustion is equal to the life time of the droplets of maximum diameter.

The figure 6 shows the evaporated volumes versus time for three values of the parameter of the distribution of the droplets injected, supposedly of Mugele Evans. It is proved that the process of combustion is practically stationary from values of τ of the order of 0.4 upwards.

2.6.2. – *Volume which has burned at each instant after the termination of the injection.*

Let us suppose two injectors exactly equal and operating in identical conditions, if at the same instant that we stop one of them we start the other, the process of combustion will be

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steady, therefore the total volume burned t seconds after the end of the injection will be given by the formula

$$g_f = \frac{\delta}{\sqrt{\pi}} \, (I - I_1) \, Gt_v$$

The figure 7 shows the end of the combustion in the three cases under consideration. In it it will be seen that the total value burned after the end of the injection is greater in the more uniform sprays ($\delta = 1.5$) than in the less uniform ($\delta = 0.5$) ones.

2.6.3. – *Combustion of a volume injected instantaneously.*

Let V be the total volume injected instantaneously. If we suppose that all the droplets begin to burn simultaneously, we get an expression of the volume burned versus time, of the form:

$$v = V \left(1 - \frac{\delta}{\sqrt{\pi}} J \right)$$

being

$$J = \int_0^{\sqrt{1-\tau}} \frac{x^4 \exp\left[- \left(\delta \ln \frac{a \, (x^2 + \tau)^{1/2}}{1 - (x^2 + \tau)^{1/2}} \right)^2 \right]}{(x^2 + \tau)^{1/2} \, (1 - (x^2 + \tau)^{1/2})} \, dx$$

The figure 8 represents graphically the said variation in the three cases under consideration.

At the beginning of the combustion the less uniform sprays burn quicker than the more uniform ones; this is due to the fact that the latter has more droplets of greater diameter, the volume of which burned in the unity of time is great. Nevertheless, the more uniform sprays burn almost completely before the less uniform ones, as can be seen in the figure, in which for $\tau = 0.4$; 99.5% was burned of the spray corresponding to $\delta = 1.5$ and only 94% of that corresponding to $\delta = 0.5$.

2.6.4. – *Superposition of successive instantaneous injections.*

In the case where before terminating the combustion of a volume previously injected, another injected volume begins with a delay of t_1 seconds, the problem is reduced to a simple superposition, as long as it is supposed that the successive injections do not modify the conditions in which those following are effected. The formula which results in this case is a follows

$$v = V \left[j + 1 - \frac{\delta}{\sqrt{\pi}} \sum_{n=0}^{n=j} \int_0^{\sqrt{1-\tau-n\tau_1}} \frac{x^4}{(x^2 + \tau - n\tau_1)^{5/2}} \, F' \left((x^2 + \tau - n\tau_1)^{1/2} \right) \, dx \right]$$

$$j \tau_1 \le \tau \le (j + 1) \tau_1$$

$j = 0.1.2.3. \ldots\ldots\ldots\ldots$

$F(x)$ is the distribution function

The figure 9 represents the fraction of the volume burned to the *total injected* in the case where five consecutive injections co-exist.

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With the usual values of the constant of combustion and in the idealized conditions under consideration in this theoretical study, it is practically impossible to achieve, with one injector alone, the interaction between two successive injections, since for this, frequencies of the order of 20.000 cycles per minute would be necessary. Nevertheless, in reality it happens that the necessary time for the droplets to reach the flame fronts is not infinitely small but that for each droplet depends on the velocity and initial sizes. As during this time the constant of evaporation of droplets is small, the mean life time of the droplets of the spray is much greater than that calculated when using values of λ of the order of those obtained by burning droplets.

3. – Scaling in Spray Combustion

3.1. – Even though considerable progress has been achieved in recent years regarding some of the fundamental processes produced in the combustion chambers of piston and turbine engines, we are still far from being in a position to apply them for the design of combustors, a difficulty which is due mainly to the great variety of elemental phenomena which are combined in the functioning of a real machine.

This state of affairs decides that, the design of the chambers or burners of a specified installation, should be based on empirical rules, on the basis of experience gathered from repeated tests, the greater the power in action the more difficult the situation, and aggravating particularly with the use of annular burners.

All this makes the use of models at reduced scales particularly attractive, from the behaviour of which can be inferred that of the burners planned by means of the application of the laws of physical similarity. In recent years, a growing interest has been given to this method, fostered by the use of turbine and jet engines for the propelling of aeroplanes and engines, in which cases operations of very high power are carried out and under very severe conditions of work.

It happens, however, that a strict application of the method is practically unattainable, because it is impossible to succeed in all the intervening dimensionless coefficients in the model and in the real burner, of which there is a considerable number, acquiring equal values. For example, S. S. Penner has shown (16) that even in the ideal case of the combustion of an isolated steady droplet, the strict physical similarity would require the equality of 23 dimensionless coefficients.

On the face of this situation, the study should move, as is usual in other processes, towards a partial physical similarity, assuming that in the change of conditions between the chamber and its model only the values of the more important coefficients are preserved. It is clear, of course, that the application of this method requires, duplicated, a previous knowledge, even through incomplete, of the essential elements of the phenomenon and, besides, that the scaling rules derived will logically be conditioned by the simplified outline of the phenomenon which has served as a basis to define the similarity.

3.2. – In such studies one must begin, in the first place, by imposing the geometrical similarity between the chamber and its model or, at least, by the similarity of its fundamental magnitudes. Supposing the said similarity has been realised, the size will be characterised by a linear dimension, the longitude or the diameter of the zone of combustion which, for example, we will call L.

In future the magnitudes corresponding to the chamber and its model will be sub-indexed as c and m respectively. Then, if n is the relation between both lengths

$$n = \frac{L_c}{L_m}$$

the said number evidently measures the geometric scale of similarity.

3.3. – In the second place, the equality of the fundamental dimensionless coefficients pertaining to the physical processes must be imposed. As in other processes, these can be of two kinds: the first, for example, such as Schmidt or Lewis-Semenov numbers, depends solely on the composition and thermodynamic state of the mixture, while the second, like Mach and Reynolds numbers, depends, besides, on the pertaining magnitudes of the movement and of the geometrical dimensions.

In some cases, an attempt has been made to draw rules of similarity by operating with different compositions or fuels in the chamber and in the model, with a view to giving greater flexibility to the test. However, the complication which this introduces is very considerable. Therefore usually both the fuel and the composition of the mixture and its temperature are ever maintained, in which case the equality of the first group of physical dimensionless coefficients is guaranteed and it is only necessary to consider the second group. In this study we will suppose also that the above conditions will be fulfilled.

The most important of the second parameters is, without a doubt, Reynolds number, since Mach number is of no interest because it is always too small in the conditions of functioning in which we are interested here. This Reynolds number Re is defined by the expression:

$$Re = \frac{\varrho VL}{\mu},$$

in where ϱ, V and μ are, respectively, the density of the mixture, the velocity of the air flow through the chamber and the coefficient of viscosity of the mixture.

Therefore, the preservation of Re requires the condition:

$$\frac{\varrho_c V_c L_c}{\mu_c} = \frac{\varrho_m V_m L_m}{\mu_m}$$

Due to the equality of composition and temperature of the mixture, the two conditions

$$\mu_c = \mu_m, \quad \frac{\varrho_c}{\varrho_m} = \frac{p_c}{p_m}$$

are fulfilled, p being the pressure.

From these relations the following condition of similarity, imposed by the equality of Reynolds number:

$$\frac{V_c}{V_m} \cdot \frac{p_c}{p_m} = \frac{1}{n}$$

is deduced.

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3.4. – In the third place the equality of the dimensionless coefficients characteristic of the chemical transformations must be assessed. Let us see which are these coefficients.

The extension of the laws of physical similarity to the phenomena in which chemical transformations are produced was performed by Damköhler in 1936 (17). The specific application of these to the processes of combustion has been studied recently, among other authors by Penner (18), who has applied besides (19), in some particular cases, the rules derived from the same. The above-mentioned works contain abundant bibliographical references to which may be added those of Weller (20) and Stewart (21) where several others are to be found.

In the study of these phenomena it has been proved that, together with the dimensionless coefficients peculiar to the physical ones, which have been dealt with before, two new coefficients appear, pertaining to the chemical transformations, introduced by Damköhler. The first of these, which we will call Da, is the relation between a measure of the time necessary to accomplish the change of composition, the so-called chemical time tq, and a measure of the time available to bring it about, the so-called residence time tr:

$$Da = \frac{tq}{tr}$$

The second dimensionless coefficient is the relation between the heat of combustion of the mixture and the internal energy of the same, or its enthalpy if the transformation is produced at constant pressure, as in the case of the turbines.

Bearing in mind that at the pressures at which they operate, both the internal energy as well as the enthalpy are independent of the pressure, and since, besides, the composition and the temperature of the mixture are preserved, the equality of the second coefficient of Damköhler is definitely guaranteed, for which it will suffice if that of the first is realized. This is, besides, fundamental, because in many cases both the possibility of ignition and that of combustion relating to a mixture depend essentially on that the chemical time is sufficiently short with respect to the residence time in order that both phenomena may be completed, a circumstance which in turn determines, according to the case, the possibility of functioning or the efficiency of the combustion.

To fix the value of the said parameter it is necessary to define, in the first place, which are the chemical and the residence times that correspond to the process under study.

Considering first the case of an isolated droplet, the chemical time tq is clearly that of its combustion or evaporation which, as was seen in the Second part of this work, is proportional to the square of the initial diameter X_o of the droplet:

$$tq = \frac{X_o^2}{\lambda}$$

As regards the residence time tr, if for example it deals with a chamber of turbine and the droplet, on reaching the zone of combustion, has the velocity V of the air in the chamber, it will be the relation between the longitude L of this and the said velocity:

$$tr = \frac{L}{V}$$

Therefore, the parameter of Damköhler is, in this case, as follows:

$$Da = \frac{X_o^2 V}{\lambda L}$$

Since λ is practically independent of the pressure, the three variables available for adjusting the value of this parameter are the size of the droplet, that of the chamber and the velocity of the passage of the air through it.

If we now go on from the isolated droplet to the spray, it will turn out that the first difficulty that arises on trying to define the chemical time, even though it evidently should correspond to the larger-sized droplets, relates to a seemingly wrongly defined and even senseless magnitude for the distribution functions different from those of Mugele-Evans. On the other hand, this magnitude is not of particular interest because what really matters is the time required for burning a given fraction of the fuel: for example, 95%, and this can be determined by knowing the Sauter diameter, a magnitude statislical clearly defined, and the distribution function, determined by its characteristical parameters.

In this connection, Figure 10 shows the influence of the shape of distribution function on the combustion time of a given fraction of spray. In it, are represented the times required to burn 90% and 95% of the fuel, referred to the time required to burn the total amount, for different values of the parameters a and δ of the distribution function set out in Chapter 1. In this is also included the variation of the Sauter diameter with the said parameters.

This figure shows how an adequate distribution of the droplet sizes enables the time of combustion of the 95% to be reduced to the fifth of total time. In particular, since increasing values of δ correspond to distributions ever more uniform, the advantage of using such sprays is clear; however, over a certain limit, when the curves tend to become horizontal, the situation cannot be improved.

Likewise Figure 11, derived from Figure 10, gives the time required to burn a given fraction of the spray, referred to that required to burn a droplet of the Sauter diameter, in function of the parameters pertaining to the distribution. The Sauter diameter has been used because it is a well defined statistical magnitude.

These considerations demonstrate how deceptive can be the study of the phenomena of evaporation and combustion of sprays, trying to define the properties of the same by one distinctive diameter, no matter what kind it may be, since, as we have seen, a more important factor than this is the time of combustion of a given fraction of the spray, and this depends, in a decisive manner, on the droplet size distribution. Unfortunately the experimental information available with regard to the characteristics of the sprays is not sufficiently complete up to now to be in a position to give rules regarding the best way to obtain such distributions, but the foregoing considerations show the advisability of continuing the investigations in this direction.

In short, the preservation of the Reynolds and Damköhler numbers with an appropriate definition of the chemical time, constitutes, as we have seen, the two conditions of similarity that determine the rules of transformation from chamber to model. To them must be added the geometrical similitude and the preservation of the composition of the mixture. Besides,

we must try to preserve the droplet size distribution function and, up to now, the necessary attention has not been given to this.

The foregoing study is based on two suppositions. In the first place, in consequence of preceding chapters, it is assumed that the important chemical time in the process is that of the evaporation or combustion of the droplets. If instead of this study, it were to be for example that of the origin of a steady flame at the base of an obstacle, a different time would have to be used (22).

As well, a different procedure would have to be applied if the fuel were to evaporate and mix entirely with the air before reaching the zone of combustion. The theoretical and experimental study on this process has been the object of various papers (23), and these are not yet sufficiently conclusive. In them, the values of the parameters deduced from experimental correlations indicate clearly the purely empirical nature of these results.

In the second place, it is assumed from the study carried out here that the spray behaviour in the atmosphere surrounding it, that is the space distribution of the droplets and its velocities, are similar in the chamber and in its model. The matter bears a relation to the problem of the injection and has also been the object of recent studies (24). In particular, the following fundamental conditions for obtaining the similarity between the mechanical behaviour of both sprays have been established: 1. Preservation of the relation between the momentum of the spray and of the air; 2. – Preservation of the angle of the spray (21). These two conditions should, therefore, be added to the previous ones, to complete the transformation rules. Nevertheless, this work requires much more study and will be the object of special consideration in a further communication.

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Fig. 1

Fig. 2

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MUGELE - EVANS	(1)	$\delta = 0.5$	$a = 1$
NUKIYAMA -TANASAWA	(2)	$\delta = 0.15$	$b = 4.08$
ROSIN - RAMMLER	(3)	$\delta = 1.61$	$\bar{x} = .63$

Fig. 3

MUGELE-EVANS	(1)	$\delta = 1$	$a = 1$
NUKIYAMA-TANASAWA	(2)	$\delta = 1.04$	$b = 11.31$
ROSIN - RAMMLER	(3)	$\delta = 3.25$	$\bar{x} = 5.61$

Fig. 4

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MUGELE -EVANS	(1)	$\delta = 1$	$a = 1.5$
NUKIYAMA -TANASAWA	(2)	$\delta = .85$	$b = 14.79$
ROSIN - RAMMLER	(3)	$\delta = 3.25$	$\bar{x} = .45$

F

$\xi \longrightarrow$

Fig. 5

$\dfrac{g_i}{G t_v}$

$\delta = 1.5$

$\delta = 1$

$\delta = 0.5$

$\tau = \dfrac{t}{t_v} \longrightarrow$

Fig. 6

$$f = \frac{v}{(j+1)\,\gamma}$$

j = 5

δ = 1.5

δ = 1

δ = 0.5

$$\tau = \frac{t}{t_v}$$

Fig. 9

Fig. 10

$\delta \longrightarrow$

1.5   1   .5   0

.20 .40 .60 .80 1.00

$\overline{z_{90}}$

$\overline{z_{95}}$

$\overline{z_{100}}$

ε_{32}

$\sigma = 1.5$

$\sigma = 1$

$\sigma = 0.5$
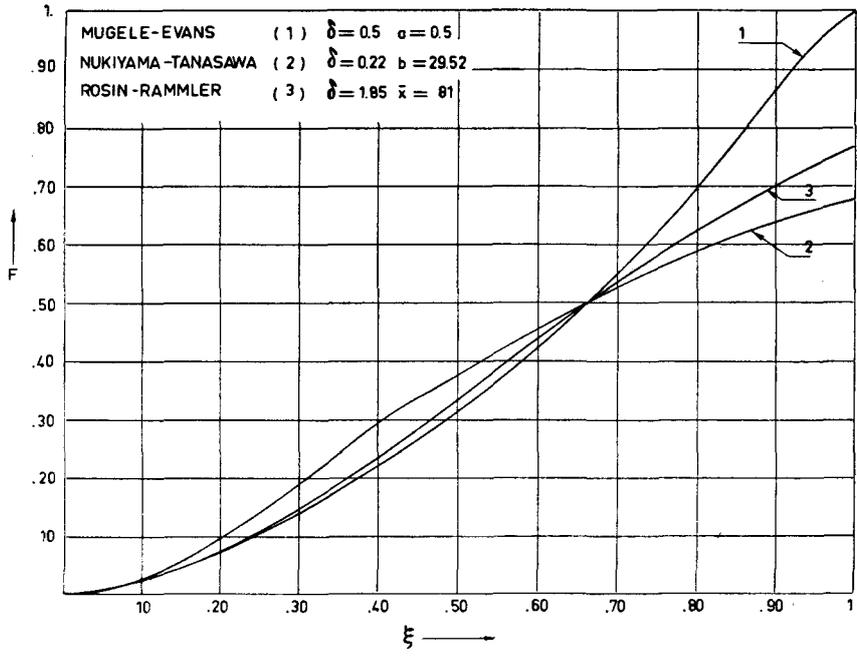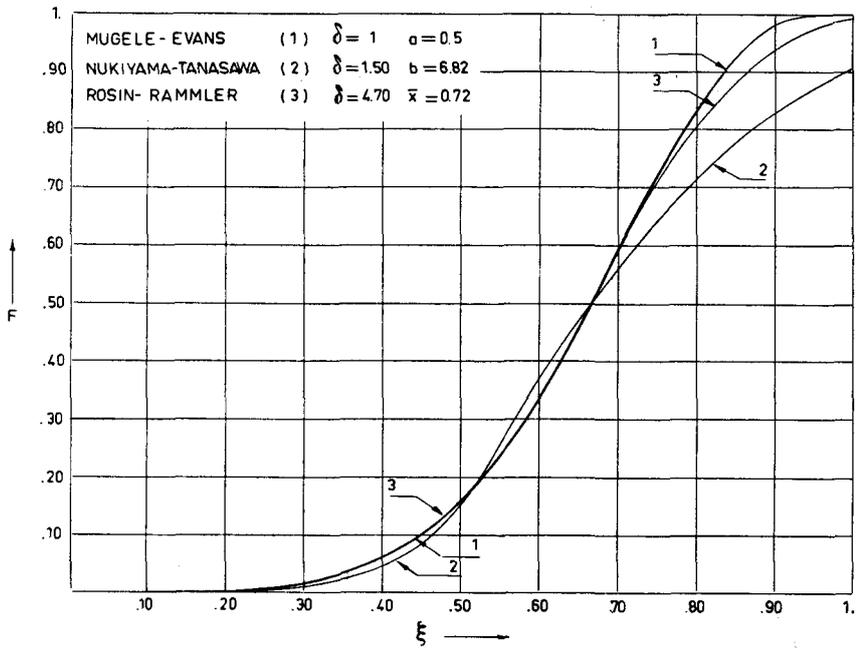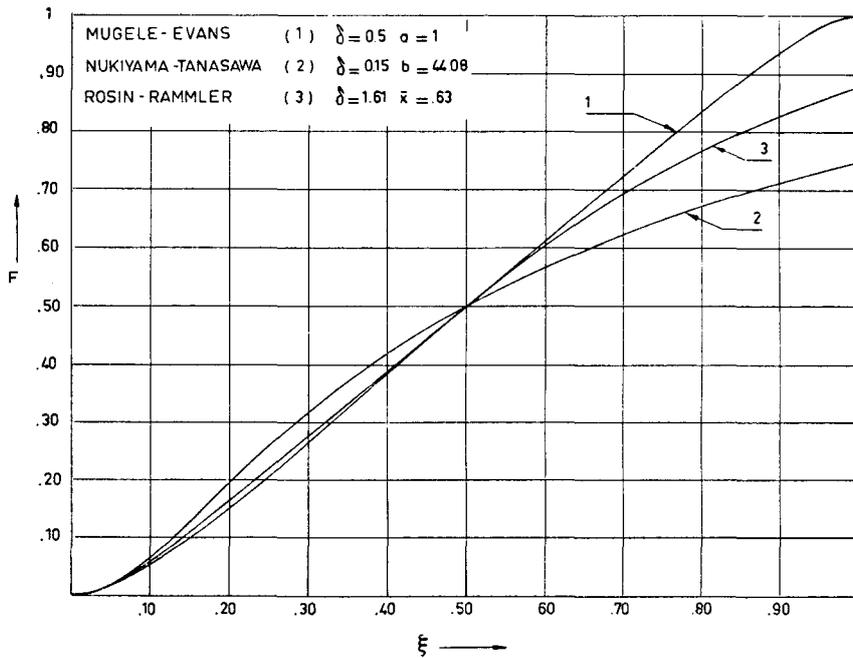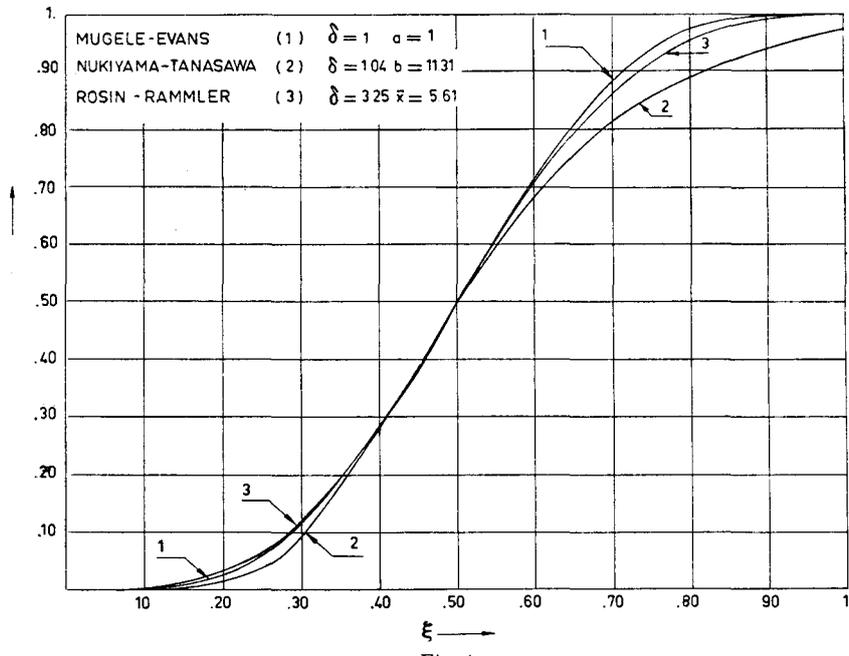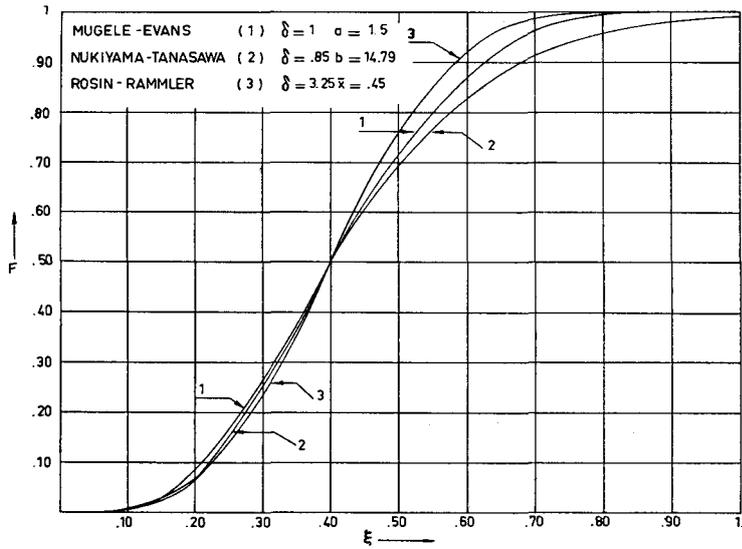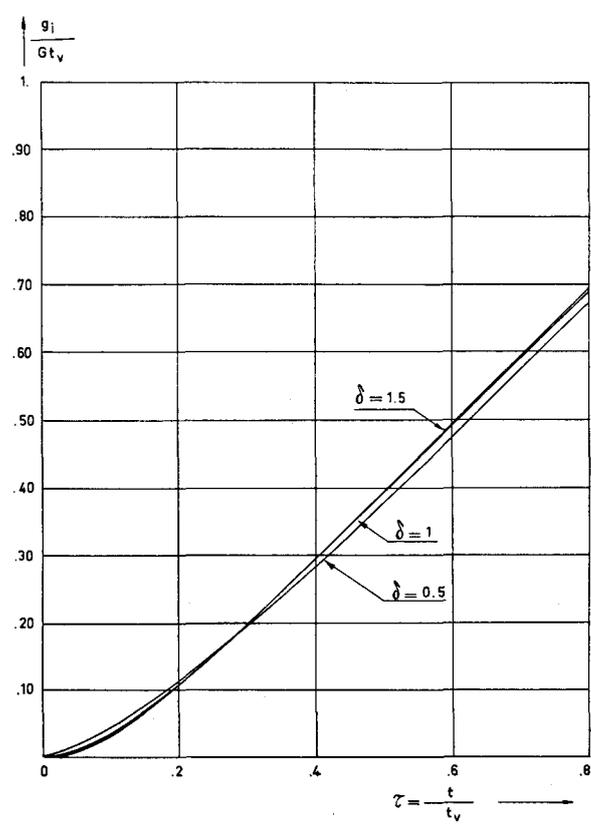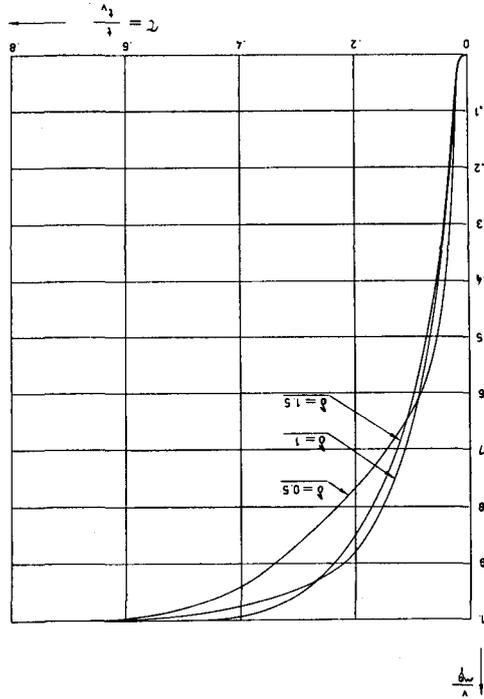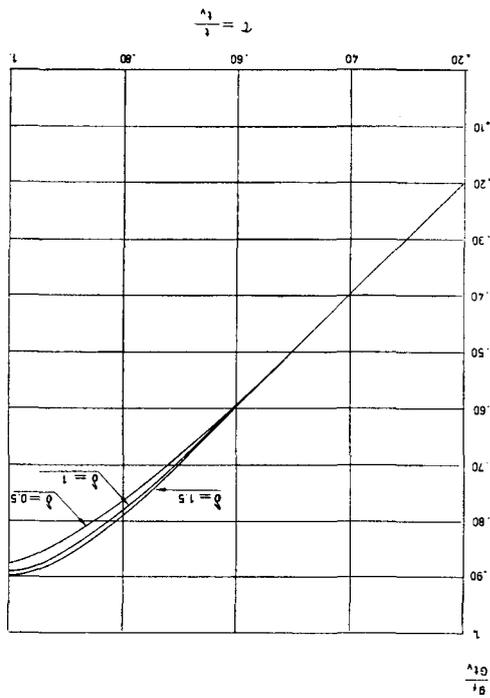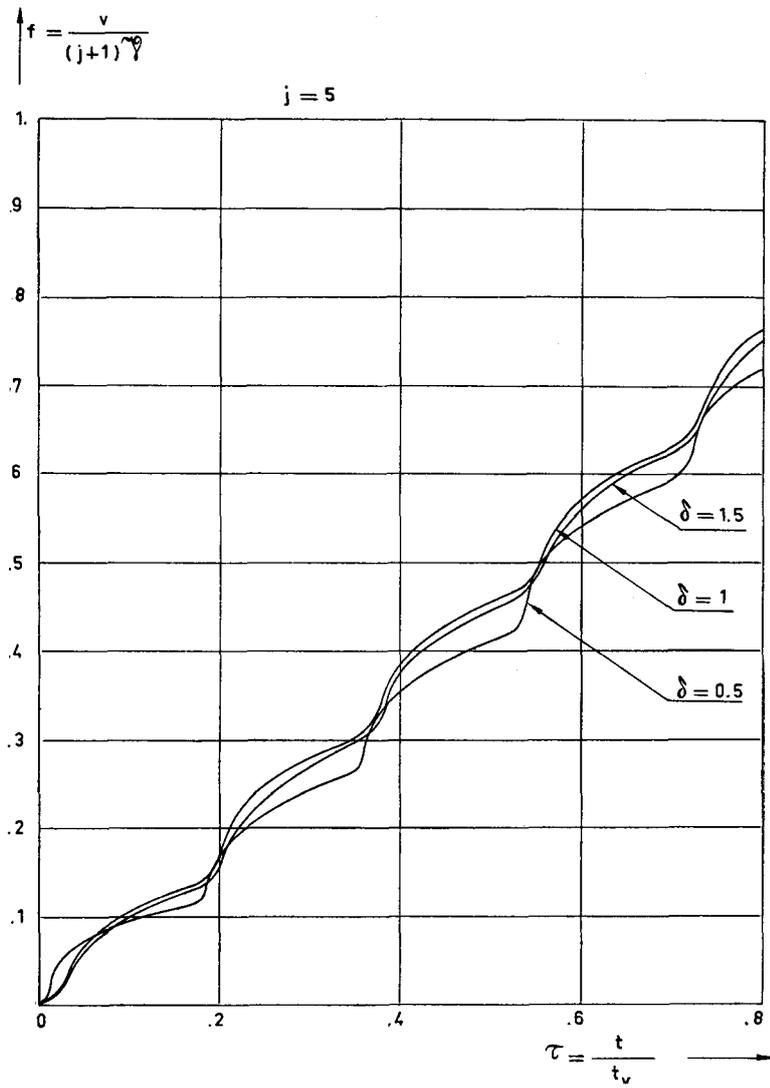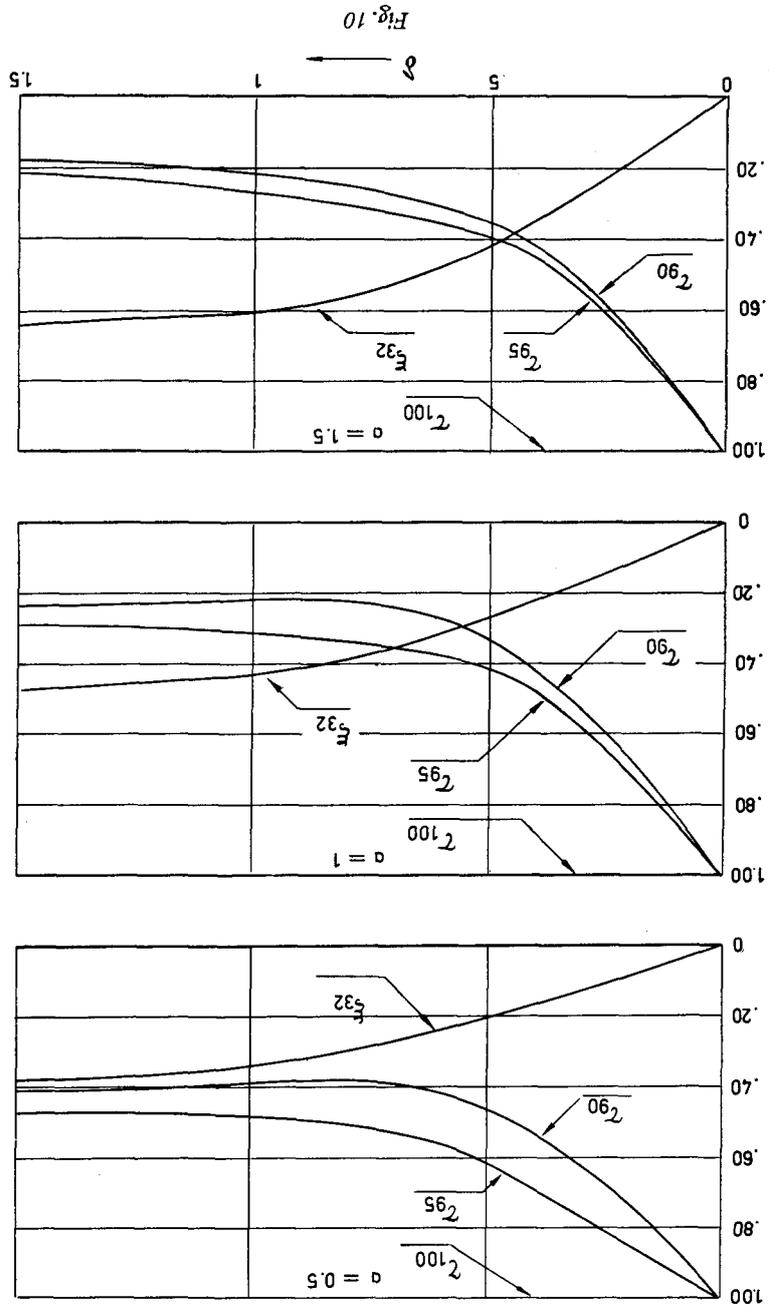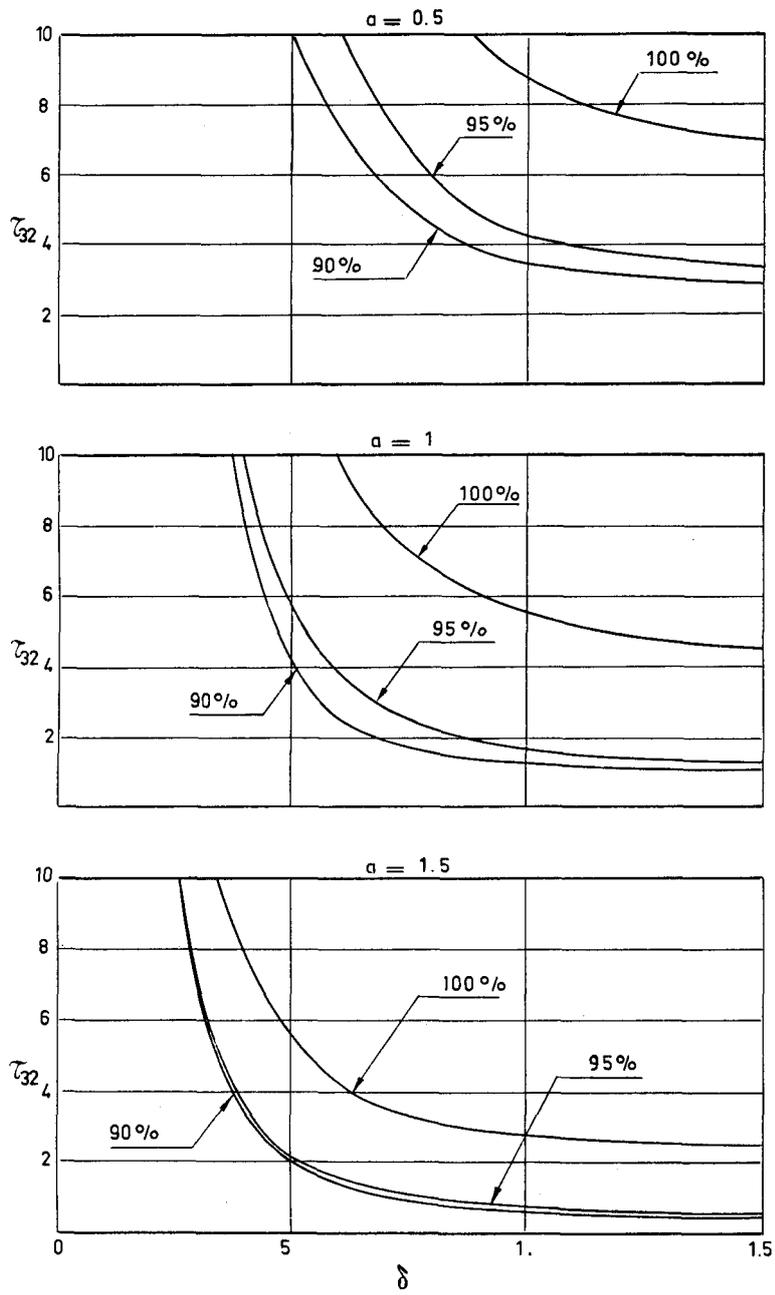
Fig. 11

DISCUSSION

Introduction

lue par M. *Sanz*, avant la discussion

L'étude actuelle résume les résultats de nos travaux dans les cas suivants:

a) Combustion au début de l'injection, cas d'une turbine à gaz.

b) Combustion à la fin de l'injection, cas d'une turbine à gaz.

c) Combustion d'un volume injecté instantanément, cas approximatif d'un moteur Diesel.

d) Superposition d'injections isolées, cas des moteurs Diesel.

Après examen des conditions de similitude dynamique appliquant le modèle de combustion de jets proposé, on expose les paramètres les plus importants du processus et d'une manière concrète l'influence de la distribution en grandeurs des gouttelettes sur le temps de combustion du jet.

Nous allons montrer en huit plaques les hypothèses et résultats les plus marquants.

Les hypothèses fondamentales sont marquées dans la plaque 1

On obtient la distribution initiale au moyen d'un récipient en rotation.

Comb. au début de l'injection.- Turbines à gaz

Fig. 1

Hypothèses fondamentales

a) $X^2 = X_0^2 - \lambda t$;
$$\begin{cases} X = \text{diam. de la gouttelette} \\ X_0 = \text{diam. inicial de la gouttelette} \\ \lambda = \text{constante de comb. ou évap.} \\ t = \text{temps de comb. ou évap.} \end{cases} (1)$$

b). Modèle de comb. = *Probert*:
Injection directe instantanée sans évap. ni mélange préalables.

c). Distrib. du jet en gouttelettes = *Mugele Evans*:
Diam. fini des gouttelettes de grand. max.

d). Procès expérimental:
Distrib. en gouttelettes égales par effet d'un récipient en rotation et photo de fractions du jet pour identifier la distrib. de particules.

Fig. 2

$$g_i = \left[\sigma - \frac{\delta}{\sqrt{\pi}}(1 - l_i) \right] Gt_v \cdot (2)$$

t_v = temps de vie d'une goutt. de diam. max.

$\delta = 1.5$
$\delta = 1$
$\delta = 0.5$

$g_i = \frac{\text{vol. brûlé dès le début de l'injection}}{Gt_v = \text{vol. brûlé pendant } t_v}$

$\sigma = \dfrac{t}{t_v} = \dfrac{\text{Temps dès le début de l'injection}}{\text{Temps de vie d'une goutt. de diam. max.}}$

On montre ici les résultats théoriques que nous avons obtenus pour le volume brûlé jusqu'à un instant quelconque de la période initiale de combustion. On peut voir que pour toute valeur de paramètre de distribution le volume brûlé est proportionnel au temps à partir de $\tau = 0,4$ environ, c'est-à-dire 0,4 du temps de combustion de la gouttelette de diamètre maximum, de façon qu'à partir de cette valeur le processus est pratiquement stationnaire.

On voit donc qu'avec le modèle et la distribution que l'on propose la période transitoire initiale de combustion est théoriquement égale au temps de vie des gouttelettes de diamètre maximum et pratiquement inférieure; sa durée étant en tout cas très réduite.

Plus le paramètre de distribution est grand, $\delta = 1,5$, c'est-à-dire, plus le jet est uniforme, plus petit est le volume brûlé dans les moments initiaux ($\tau < 0,2$), mais on observe tout de suite des valeurs de volumes brûlés plus élevées avec des jets uniformes ($\tau > 0,2$).

On représente ici le problème concernant la phase finale de combustion succédant à celle d'injection.

Le processus se maintient pratiquement stationnaire jusqu'à 0,5 du temps de vie de la gouttelette de grandeur maxima après la fin de l'injection.

A partir de ce moment et pour des jets plus uniformes, le volume brûlé est plus grand dans la même période de temps qu'avec des jets moins uniformes.
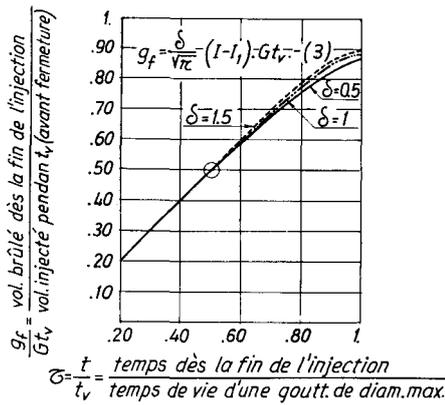
Fig. 3
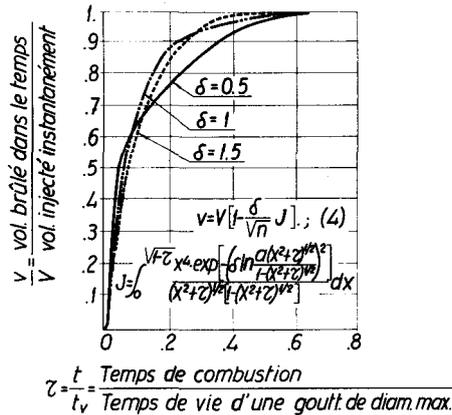
Fig. 4

Supposant que l'on injecte instantanément un volume V et que toutes les gouttelettes commencent à brûler en même temps, cas qui pourrait s'appliquer à la combustion dans un moteur Diesel, on a calculé le volume brûlé.

On peut très bien voir qu'en tout cas le 75% du jet brûle dans notre diagramme de combustion avant 0,2 du temps de vie des gouttelettes de diamètre maximum et le 100% pratiquement à 0,6 du même temps.

Autre importante particularité que l'on a remarquée consiste dans le fait qu'au début les jets très uniformes brûlent plus lentement que ceux qui le sont moins, alors que le 100% brûle avant.

Ce résultat peut être intéressant en ce qui concerne l'avance à l'injection dans les moteurs ou dans la combustion prolongée.

On montre ici le cas d'injections instantanées superposées et leurs interactions. On a pris un temps t_1 entre deux injections consécutives et supposé que les conditions des suivantes ne varient pas.

Le graphique représente 5 injections consécutives.

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Comb. de 5 injections instantanées
superposées, décalées de t_i sec.

$$v = V \left[6 - \frac{p}{V n} \sum_{n=0}^{n=5} \int_{0}^{\sqrt{z-nz_i}} \frac{x}{(x^2+z-nz_i)^{3/2}} F''\left((x^2+z-nz_i)^{1/2}\right) dx\right] (5)$$

$$\tau = \frac{t}{t_v} = \frac{\text{Temps de combustion}}{\text{Temps de vie d'une goutt.de diam. max}}$$

Fig. 5

Pour les valeurs courantes de la constante de combustion que nous avons présentées au Congrès de Zürich, il est pratiquement impossible qu'avec un seul injecteur il y ait interaction dans un moteur Diesel, du fait qu'il faudrait pour cela 20.000 cycles/minute théoriquement, ce qui pourrait éventuellement se produire avec plusieurs injecteurs.

Dans ce cas on devra, avec les formules déduites, effectuer une étude à soi pour chaque type de moteur, car il faut compter avec le temps que la gouttelette met avant d'arriver au front de flamme, qui n'est pas négligeable, étant lié à la vitesse et à la grandeur de la gouttelette elle-même, de façon que pendant cette période la constante de combustion est plus petite, le temps de vie des gouttelettes étant plus long.

Conditions de similitude dynamique
Paramètres physiques :
a)Compos. identique du combust.dans la chambre et le modèle; c'est-à-dire
b)Tempér. "   "   "   "   "   "   "
  Numéros de Schmidt, Lewis-Semenov, etc., les mêmes
c)Numéros de Reynolds,doivent être égaux

$$\frac{V_c}{V_m} \cdot \frac{P_c}{P_m} = \frac{1}{n} \;,\quad (6)\;,\quad \text{où:}$$

V_c = vit. dans la chambre; V_m = vit. dans le mod.
P_c = press. "   "   "   ; P_m = press. "   "   "
n = simil. géom. lin. = $\dfrac{\text{long. chambre}}{\text{long. modèle}}$

Paramètres chimiques :
d)Deuxième par. de Danköhler = $\dfrac{\text{chal. comb. mél.}}{\text{énergie int. mél.}}$
  se vérifie pour (a)et (b).
e)Premier par. de Danköhler : $D_a = \dfrac{X_o^2 \cdot V}{\Lambda \cdot L}$; $\quad(7),\;$ où:
X_o=Diam.max. de la gouttelette
V = Vit. de l'air dans la chambre
Λ=Constante de comb. moyenne
L = Longueur de la chambre

Fig. 6

Temps de comb. d'un pour-cent de jet par rapport au temps de comb. du 100% du jet

δ = Par. d'uniformité du jet

Fig. 7

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Les paramètres physiques et chimiques pour les conditions de similitude dynamique on montre ici.

Ce qui importe pratiquement est connaître le temps qu'il faut pour qu'une certaine quantité de combustible, 90% par example, brûle et en ce cas le diamètre choisi est celui de Sauter, bien défini dans quelconque distribution.

Le graphique montre comme la forme de la fonction de distribution puisse influencer le temps de combustion d'une fraction de jet.

On a indiqué en abscisses les valeurs de δ, paramètre de distribution, pour trois différentes valeurs de a, diamètre moyen des gouttelettes.

En ordonnées on a représenté les temps sans dimension τ pour brûler 90%, 95% et 100% du jet.

On voit que dans le cas où a = 1,5 et δ = 1,5 on peut obtenir des temps pour 95% du jet d'un cinquième plus bas que pour la combustion totale.

Un moment arrive toutefois où la courbe reste horizontale et on n'obtiendrait aucun effet pratique ultérieur en améliorant la distribution du jet.

Cela prouve qu'il faut continuer les recherches jusqu'à obtenir une distribution plus adéquate.

Le graphique montre également en ordonnées le diamètre Sauter correspondant.

Temps de comb. d'un pour-cent de jet par rapport au temps de comb. d'une goutt. de diam. Sauter
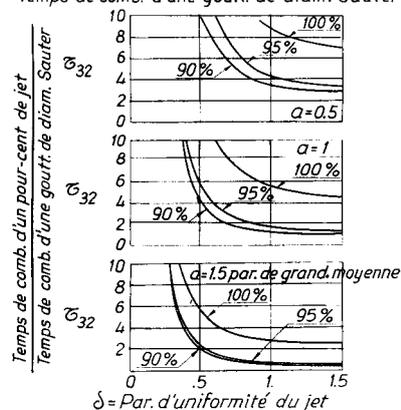
Fig. 8

Le graphique montre les temps pour brûler une fraction donnée de jet, par rapport au temps qu'il faut pour brûler une gouttelette de diamètre égal à celui de Sauter, en fonction des paramètres d'uniformité δ, en abscisses, et de a, grandeur moyenne.

On peut arriver, avec des valeurs de jets uniformes, à brûler 95% du jet en moitié de temps Sauter, sans obtenir, même en augmentant l'uniformité du jet, d'aller pratiquement au dessous de cette valeur.

Pour conclure, nous estimons que pour l'étude de la similitude dynamique il faut non seulement établir les Numéros Reynolds et Danköhler, adaptant convenablement le temps chimique au modèle de combustion en tout ce qui concerne injection directe dans la chambre, stabilisation par obstacle, évaporation-mélange-combustion, etc., mais retenir aussi la fonction de distribution de gouttelettes en grandeurs, ainsi que nous venons de le dire.